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THE CENTRAL IDEA OF MODELING – OPTIMIZATION IN REAL-LIFE-SITUATIONS

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Resumen

En los últimos años la idea central de modelización fue discutida intensivamente en varios paises por varios pedagogos de matemática e informática. Con la ayuda de esta idea fundamental es posible implementar problemas de la vida real en el proceso científico de aprendizaje. Por lo tanto, la matemática se vuelve más fascinante y los estudiantes se sienten más motivados. Muchos pedagogos intentaban implementar ejemplos de optimización con la ayuda de Computer Algebra Systems (CAS), y muchas ideas para implementarlos con CAS, ya existen. En los últimos cinco años los software de geometría dinámica fueron utilizados cada vez más en educación, principalmente para mostrar situaciones gráficas de una manera sencilla. En este artículo quiero mostrar cómo es posible tratar problemas de optimización -otra idea central bien aceptada- con estudiantes, por la combinación de software de geometría dinámica con CAS, utilizando sus ventajas particulares en situaciones específicas. La estructura básica del artículo va a ser la idea fundamental de modelización, la cual contribuye a mostrar la necesidad de problemas de optimización, los cuales deberían ser discutidos intensamente en las escuelas.

Palabras clave

Modelización - Optimización - Software de geometría dinámica - CAS - Aprendizaje.

Summary

In the last few years the central idea of modeling was discussed intensively in several countries by different Mathematics and Informatics educators. With the help of this fundamental idea it is possible to implement real-life problems in the natural-science teaching process. Therefore Mathematics is getting more fascinating and students are getting more motivated. A lot of educators have tried to implement optimization examples with the help of Computer Algebra Systems and a lot of ideas for implementing such examples with CAS exist. In the last five years dynamical geometry software, was used more and more in education, mainly because it is to demonstrate graphical situations in an uncomplicated way. In this paper I want to show how it is possible to discuss problems of optimization – another well-accepted central idea – with students through the combination of dynamical geometry software and Computer Algebra Systems using their specific advantages in certain situations. The framework of the paper will be the fundamental idea of modeling which helps to present the necessity of problems of optimization in schools and should be discussed there intensively.

Key Words

Modeling - Optimization - Dynamical geometry software - CAS - Teaching process.

Introduction

Problems in real life, like problems in environment, problems in sports or problems in traffic, are often the starting point calculations in mathematics or application of mathematics. But for a mathematical discussion it is necessary that students are able to understand the given problem. With the help of the central idea of modeling in education it is possible that students get an additive so that they are able to take a schedule for discussing difficult problems. Werner Blum, a German Mathematics educator, has occupied himself with the topic of modeling for students since twenty years. Because of the implementation of educational standards in German speaking countries the central idea of modeling was highlighted again. In the development of educational standards it turned out that modeling should be a main basement which students are able to do when their mathematics education is finished. This fact is not very astonishing because one of the main businesses of Mathematics is the intervention in general education. Trough the help of modeling this intervention could be done. Therefore Hans-Joachim Vollrath (2001) has quoted four arguments to do modeling in education:

- Development of personality.
- Development of environment.
- Equal participation in (all aspects of) society.
- Agency of standards.

Modeling can also be sought in curricula. It can be found in a lot of paragraphs, e.g. in the Austrian curriculum for Mathematics (2004): "Mathematics in education should contribute that students are able to enforce their accountability for lifelong learning. This can happen through an education to analytical-coherent thinking and through intervention with mathematical backgrounds which have a necessary fundamental impact in many areas of life. Acquiring these competencies should help students to know the multifaceted aspects of mathematics and its contribution to several different topics.

The description of structures and processes in real life with the help of Mathematics allows understanding coherences and the solving of problems through a deepened resulting access which should be a central aim of Mathematics education. [...]"

Learning in application-oriented contexts

An application-oriented context points out the usability of Mathematics in different areas of life and motivates to gain new knowledge and skills. Integration of the several inner-mathematical topics should be strived in Mathematics and through adequate interdisciplinary teaching-sequences. The minimal realization is broaching the issue of application-oriented contexts in selected mathematical topics; the maximal realization is the constant addressing of application-oriented problems, discussion and reflection of the modeling cycle regarding its advantages or constraints."

If we look at the listed points above and the curricula paragraphs the model which Blum (2005) has designed will be obviously:

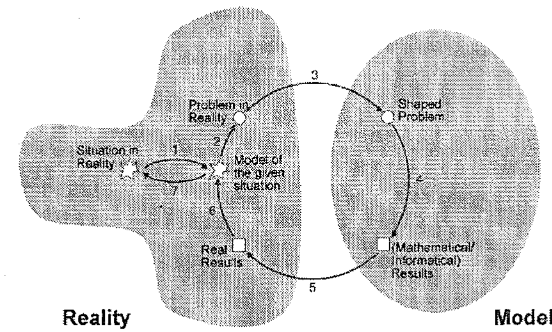


Figure 1. Modeling cycle, designed by Blum (2005)

The figure could be explained like this:

First we have a given situation in reality, which has to be understood (1). A suitable model for the given situation is constructed. The parameters of this suitable model should be reduced (2) so that a workable model which describes reality is born. This model can be illustrated through (mathematical) algorithms (3). The shaped problem could be solved by mathematical or informatical strategies (4). Then the results need to be re-interpreted to reality (5) if they are good enough and have to be validated (6) and argued (7). But if any step is not done well enough, so that it is obvious that the model is not good enough for the given problem it will be possible to return to the model of the situation, sharpen it and run through the circuit again.

Modeling with the help of technology

Through the usage of computers in education it is possible to discuss problems which can be taken out of the life-world of students. Through such discussions the motivation for Mathematics education can be effected because students recognize that Mathematics is very important in everyday life. If it is possible to motivate students in a way like this it will be easy to discuss and to teach necessary basically or advanced mathematical contents, like finding a function or calculating the local extreme of a function.

Unfortunately a lot of educators do not think in this way. The reasons for it a manifold, e.g. educator do not want to use CAS or other technology in class or the preparation for such topics is very costly in terms of time. But such arguments will not be very effective if we want to motivate students for Mathematics. Karl Fuchs and Werner Blum (2008) quote aims of Jutta Möhringer (2006) which can be reached through (complex) modeling with technology:

- Pedagogical aims: With the help of modeling cycles it is possible to connect skills in problem-solving and argumentation. Students are able to learn application competencies in elementary or complex situations.
- Psychological aims: With the help of modeling the comprehension and the memory of mathematical contents is supported.
- Cultural aims: Modeling supports a balanced picture of Mathematics as science and its impact in culture and society (see Maaß (2005a), Maaß (2005b))

- Pragmatically aims: Modeling problem helps to understand, cope and evaluate known situations.

As we can see the use of technology can help to simplify the difficult procedures in modeling. In some points the use of technology is indispensable:

- Computationally-intensive or deterministic activities.
- Working, structuring or evaluating of large data-sets.
- Visualizing processes and results.
- Experimental working.

With technology in education it is possible to teach traditional contents different to common methods and it is very easy to pick up new contents for education. The center of education should be a discussion with open, process-oriented examples. They are characterized by the following points. Open process-oriented problems are examples which...

- ... are real applications, e.g. betting in sports (Siller, Maaß (2008)), not vested examples for mathematical calculations;
- ... are examples which develop out of situations, that are strongly analyzed and discussed;
- ... can have irrelevant information , that must be eliminated, or information which must be found, so that students are able to discuss it;
- ... are not able to solve at first sight. The solution method differs from problem to problem;
- ... need competencies not only in Mathematics. Other competencies are also necessary for a successful treatment;
- ... motivates students to participate;
- ... provokes and opens new questions for further as well as alternative solutions.

The teacher achieves a new role in his profession. He is becoming a kind of tutor, who advises and channels students. The students are able to detect the essential things on their own.

Let us have a look at a given example which is well known in classic Mathematics education – the optimization of the surface of a cylindrical object, when the volume is given.

Example

Through the usage of technology in education it was getting easier to discuss examples that could be found in real-life. Although the level of difficulty was rising, a lot of educators are motivated for discussing such examples.

In this article I do not want to discuss such difficult examples; I want to show the possibility of dynamic graphical software and its advantages to students compared to traditional solutions. Let us have a look at a traditional example which could look like the following:

Didi Mateschitz the owner of the Red-Bull-company has looked at his costs for producing tins in which the soft-drink is filled in. He recognizes that the costs are rising to a limit which he is not able to fund. So he is looking for alternatives and asks a student of Mathematics for a competitive solution. Think about the answer the mathematician will give to Mateschitz!

- *First step: Constructing a suitable model for the given situation*

The given situation is a "real situation" of reality. Didi Mateschitz wants to optimize his production costs for the tins in which his soft drink is filled.

First of all the student will look at the volume of such tins. Therefore he walks to a supermarket and buys on tin of the soft-drink (shown in the picture besides). He looks at the

tin and discovers the volume of this tin: V = 250 ml. Then he thinks about the described situation and realizes that he has to think about the surface of this tin. He thinks to himself: If this surface is optimized then he can hardly do anything. But if this surface is not optimized then he will succeed.

- *Second step: Reducing the parameters*

The student recognizes that the form of the tin is a little bit complicated for an elementary calculation. The seam at the top and the recess at the bottom are too difficult for an easy explanation. So he decides to idealize the tin. The idealized tin looks like a cylinder. There are no seams, no recesses and no break contact at the top. With the help of this idealized tin he is able to calculate the optimized surface.

- *Third step: Constructing a mathematical modeling*

Because of his knowledge the student describes the surface of the tin with the help of a function in several variables:

$$S(r,h) = 2r^2\pi + 2r\pi h$$

But he is not able to solve such a function with elementary calculations. So he takes the volume as a constraint:

$$250 = r^2\pi h$$

In the next step he transforms this equation to $h = \frac{250}{r^2\pi} h = \frac{250}{r^2\pi}$ and substitutes *h* in the function *S(r,h)*. Thereby he composes a function in one variable *S(r)*, which he is able to solve:

$$S(r) = 2r^2\pi + \frac{500}{r}$$

- *Fourth step: Solving the function*

Now the student will calculate the solution of the function with the help of differential calculation. If he is thinking in pictures maybe he will draw first of all the graph of the function so that he can compare the calculated and the graphical solutions. Maybe he will do this with a graphical calculator or a computer algebra system (CAS) – as I did it in this article with Derive:

Calculated solution

#1: $S(r, h) := 2 \cdot r^2 \cdot \pi + 2 \cdot r \cdot \pi \cdot h$

#2: $250 = r^2 \cdot \pi \cdot h$

#3: $S(r) := 2 \cdot \pi \cdot r^2 + \frac{500}{r}$

#4: SOLVE(S'(r), r)

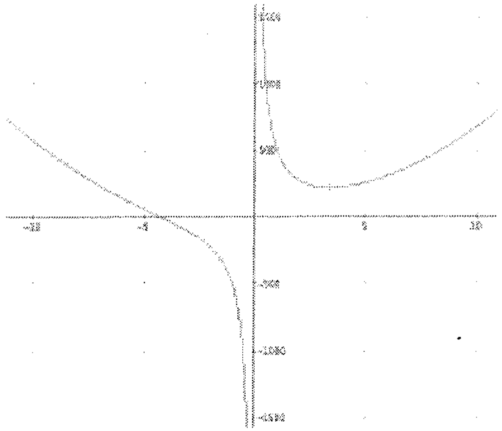
#5: $r = -\frac{5}{2 \cdot \pi^{1/3}} - \frac{5 \cdot \sqrt{3} \cdot i}{2 \cdot \pi^{1/3}} \vee r = -\frac{5}{2 \cdot \pi^{1/3}} + \frac{5 \cdot \sqrt{3} \cdot i}{2 \cdot \pi^{1/3}} \vee r = \frac{5}{\pi^{1/3}}$

#6: $h = \frac{10}{\pi^{1/3}}$

#7: $s\left(\frac{5}{\pi^{1/3}}, \frac{10}{\pi^{1/3}}\right)$

#8: $150 \cdot \pi^{1/3}$

Graphical solution



The minimum can be found graphical at the cross position, which is marked by the ellipse.

With the help of dynamic graphical software a tool was developed which is able to calculate the solution and to show the solution in one step. The student has to know the software well and then he is able to implement this problem with the help of such software. The software I am taking for such "experiments" is GeoGebra, which can be downloaded at www.geogebra.at. There are some advantages to other dynamic graphical software:

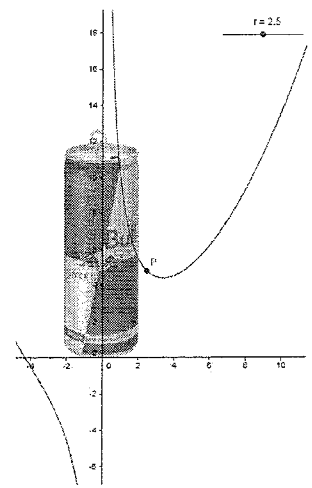
- It is free of charge.
- The algebra and the graphic window are side by side.
- It is easy to put in dynamical background images.
- The implementation of scroll bars is easy for students.

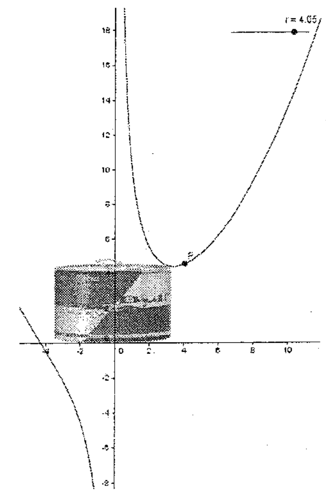
In the first step the student has to implement a scroll bar because if he does this in the first step the worksheet is dynamical from the very first beginning. Then the background image can be implemented and three points of this image should be described trough coordinates. Those can be chosen on our own; a good description could be the following: Point 1 *(-r, 0)*, Point 2 *(r, 0)* because the base of the cylinder

is a circle; Point 3 could be described through *(-r, h)*, where *h* must be specified for some reasons. The main reason for the specific description of Point 3 is the dynamical aspect of the background image. Because if *h* is described through the equation which we got through the constraint the background image is described only through the help of our parameter *r*. The explained situation can be seen in the following picture.

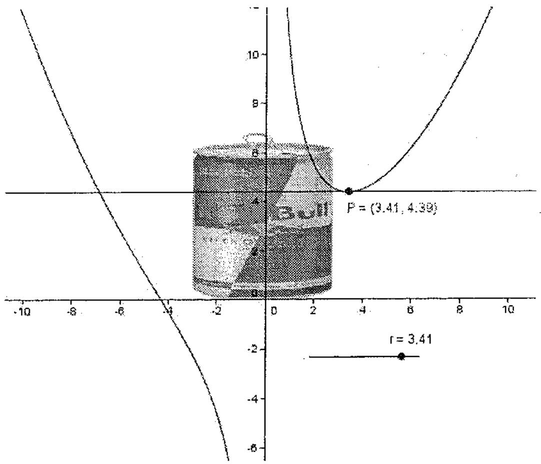
This figure shows a tin in reality very proportional because the base circle of such a tin is about 5 cm and the height is about 13 cm. Looking at the values of the coordinates shows that these values which were constructed are nearly the same. The aberration in the height is explained easily: the pictured tin has borders. Because of the bended up break contact the borders are displaced. For this reason we have to work with another idealized situation. But this does not matter because for demonstrating the dynamical aspect the presented situation is enough.

After creating this starting point the points A, B and C are hidden and the function is implemented in GeoGebra. Another important point is the implementation of point P *(r, f(r))*. This can be seen in the left picture. If the scroll bar is moved now the presentation in this left picture will change, maybe to the presentation shown in the right picture. The alteration of the values of *r* can be seen at the scroll bar, in the right upper corner.





As we know the gradient of the tangent is 0 when the minimal surface is reached. Therefore the tangent in the point *P* is sketched and the minimal surface is searched for. It can be found at the point *P* (3.41, 4.39) which is not very surprisingly because we have calculated this before.



- *Fifth step: Interpreting and arguing the solution*

Through a detailed view to the results of the radius and the height of the tin it is easy to see, that the minimal surface is reached when the height is equal to two times the radius (*h=2r*), which can be shown trough a simple analogue calculation – let us name it a pseudo-proof – with the help of the CAS Derive:

#1: $O(r,\ h) := 2\cdot r^2\cdot\pi + 2\cdot r\cdot\pi\cdot h$

#2: $h = \dfrac{v}{r^2\cdot\pi}$

#3: $O(r) := 2\cdot r^2\cdot\pi + \dfrac{2\cdot r\cdot\pi\cdot v}{r^2\cdot\pi}$

#4: $\dfrac{d}{dr}\left(2\cdot\pi\cdot r^2 + \dfrac{2\cdot v}{r}\right)$

#5: $4\cdot\pi\cdot r - \dfrac{2\cdot v}{r^2}$

#6: $\mathrm{SOLVE}\left(4\cdot\pi\cdot r - \dfrac{2\cdot v}{r^2},\ r\right)$

#7: $r = v^{1/3}\cdot\left(-\dfrac{2^{2/3}}{4\cdot\pi^{1/3}} - \dfrac{\sqrt{3}\cdot 2^{2/3}\cdot i}{4\cdot\pi^{1/3}}\right) \vee r = v^{1/3}\cdot\left(-\dfrac{2^{2/3}}{4\cdot\pi^{1/3}} + \dfrac{\sqrt{3}\cdot 2^{2/3}\cdot i}{4\cdot\pi^{1/3}}\right) \vee r = \dfrac{2^{2/3}\cdot v^{1/3}}{2\cdot\pi^{1/3}}$

#8: $h = \dfrac{2^{2/3}\cdot v^{1/3}}{\pi^{1/3}}$

#9: $\dfrac{h}{r}$

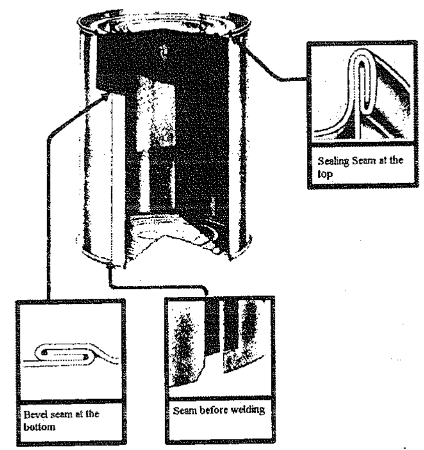
#10: 2

- *Sixth step: Validating the solution*

As we have seen the minimal surface of such a tin, which can be compared to a cylinder, can be found when *h* equals *2r*. Now students could search for such tins for instance in a supermarket. They will soon find such tins. My thesis for finding such surface optimized tins is the following: "If the design is not important to the producers the surface is optimized. If the design or maybe the image of a company is important then the tins are not optimized." I have not proofed this thesis but if we have a look at tins in the supermarket it gets obvious. Tins in which drinks are kept are not surface-optimized; tins where eatables are kept, like tins for vegetables, especially (sweet-) corn, are surface optimized, because the design is not important to the buyer. If we have a look at the pictured corn-tin my conjecture will be reasonable.

Prologue

As we have seen the situation of modeling a Red-Bull tin and calculating the minimal surface when the volume is given was idealized very much. In a first modeling cycle this is a proper approach. But for sure students will ask what they have to do, if other parameters like the seam should be considered. For this reason a CAS should be taken, because it is difficult to implement such conditions with the help of dynamical graphical software. For a better understanding of tins in reality it is necessary to discuss tins with seams with students. Let us have a look at the following picture of Deuber (2005) which can be found in the internet:



Through the help of the picture it is obvious to see that more material is needed than our calculation has shown, because considering the seam means enlarging the surface. Therefore the function describing the surface has to be changed. The solution and a detailed description for this problem can be found at http://mat.mued.de/lager/ue/ue-an/an-04-32.zip. The author describes the enlarging of the surface very well:

- Call 1: "The beading edge at the top and the bottom of a tin needs 7-8 mm more material; and another important claim in this context is the consistency of the edge, which is demanded by "minimum 2 mm height".
- Call 2: The seam of the nape for the beading nape needs 3-4 mm more material for each top; the claim of consistency must be fulfilled too.
- Call 3: For closing the seam at the nape another addition of 7.5 mm is needed."

With the help of all these additions to the surface the function has to be changed to:

- The surface of the circles at the top and the bottom changes because of call 1 for about 15mm:

$$O(r,h) = 2r\pi h + 2(r+7.5)^2\pi \quad O(r,h) = 2r\pi h + 2(r+7.5)^2\pi$$

- The surface of the nape changes because of call 2 for about 7 mm:

$$O(r,h) = 2r\pi(h+7) + 2(r+7.5)^2\pi \quad O(r,h) = 2r\pi(h+7) + 2(r+7.5)^2\pi$$

- Through the addition of call 3 the function of the surface changes another time and the "new" objective function is found:

$$O(r,h) = (2r\pi + 7.5)(h+7) + 2(r+7.5)^2\pi$$

With the help of the constraint $h = \dfrac{V}{r^2\pi} h = \dfrac{V}{r^2\pi}$ it is possible to solve the objective function depending to the variable *r*. The implementation in Derive could look like the following (Volume = 425 ml, which meets a tin of corn in reality):

#1: $O(r, h) := (2\cdot r\cdot\pi + 7.5)\cdot(h + 7) + 2\cdot(r + 7.5)^2\cdot\pi$

#2: $h = \dfrac{v}{r^2\cdot\pi}$

#3: $\dfrac{4\cdot\pi^2\cdot r^4 + 88\cdot\pi\cdot r^3 + 15\cdot\pi\cdot r^2\cdot(15\cdot\pi + 7) + 4\cdot\pi\cdot r\cdot v + 15\cdot v}{2\cdot\pi\cdot r^2}$

#4: $O(r) := (2\cdot r\cdot\pi + 7.5)\cdot\left(\dfrac{v}{r^2\cdot\pi} + 7\right) + 2\cdot(r + 7.5)^2\cdot\pi$

#5: $\dfrac{d}{dr}\, O(r)$

#6: $\dfrac{4\cdot\pi^2\cdot r^4 + 44\cdot\pi\cdot r^3 - 2\cdot\pi\cdot r\cdot v - 15\cdot v}{\pi\cdot r^3}$

#7: $\dfrac{4\cdot\pi^2\cdot r^4 + 44\cdot\pi\cdot r^3 - 2\cdot\pi\cdot r\cdot 425000 - 15\cdot 425000}{2\cdot\pi\cdot r^3}$

#8: $\text{SOLVE}\left(\dfrac{4\cdot\pi^2\cdot r^4 + 44\cdot\pi\cdot r^3 - 2\cdot\pi\cdot r\cdot 425000 - 15\cdot 425000}{2\cdot\pi\cdot r^3}, r\right)$

#9: $\text{APPROX}\left(\text{SOLVE}\left(\dfrac{4\cdot\pi^2\cdot r^4 + 44\cdot\pi\cdot r^3 - 2\cdot\pi\cdot r\cdot 425000 - 15\cdot 425000}{2\cdot\pi\cdot r^3}, r, \text{Real}\right)\right)$

#10: $\dfrac{2\cdot(\pi^2\cdot r^4 + 11\cdot\pi\cdot r^3 - 212500\cdot\pi\cdot r - 1593750)}{\pi\cdot r^3}$

Derive is not able to calculate an algebraic solution! Therefore expression #10 is solved numerically. First without borders, but as the solution is not adequate to our problem we have to seek other solutions.

#11: $\text{NSOLVE}\left(\dfrac{2\cdot(\pi^2\cdot r^4 + 11\cdot\pi\cdot r^3 - 212500\cdot\pi\cdot r - 1593750)}{\pi\cdot r^3}, r\right)$

#12: $r = -2.389060038$

#13: $\text{NSOLVE}\left(\dfrac{2\cdot(\pi^2\cdot r^4 + 11\cdot\pi\cdot r^3 - 212500\cdot\pi\cdot r - 1593750)}{\pi\cdot r^3}, r, 100, 10\right)$

#14: $r = 38.21578575$

As we can see the solution of our calculation is $r = 3.8$ cm. If we compare the solution of this calculation to the solution of a tin which was modeled without seams it is obvious to see that the solution differs very much (the radius of the material for the circle at the bottom and the top has to be increased for about 11.43% - the other values can be evaluated in a same way); so a lot more material is needed for a tin in reality.

Conclusion

In summary we may see that constructing functions is a central idea in the topic of modeling. With the structuring principles of functions it is possible that a lot of important problems can be treated with different software systems, like CAS or DGS. Therefore examples of reality have to be treated more in education so that students are aware of the necessity of Mathematics and the use of technology in it.

As Schneider (1999) tells in her paper "Analysisausbildung mit dem Computer" (in English: Teaching Analysis with computers). It is necessary to think about the role and the significance of operations and computer science (Relation: Activity – Science). Especially the center of Mathematics should be in the teaching of mathematical concepts. Through using interactive dynamical systems it is possible that special concepts in optimization are taught very descriptive.

With the help of dynamical visualization it is possible to discuss examples in a new (intuitive) understandable way. Cognitive association can be created as Meissner (1996) has mentioned: "Through attracting the attention of pupils on the essential points a (cognitive) schemata of activity is developed. Exactly this schema is building the skeletal structure of the new idea. An important role in this process of individual acquirement of concepts are mental images, so called 'prototypes' […] while working with visualized descriptions when acquiring concepts those mental images should be activated and the learning of those concepts should be lightened." So technology in education is giving us the chance to handle activities at a new level. Through the preparation of technology in education tools for demonstrating, for discussing examples at a new level – not only calculations with a lot of computing time or error-prone problems – and tools for self-experienced activities of students in numerical, graphical, symbolical and dynamical areas are provided and prepared.

By integrating technology in education routine jobs, like differentiation or integration of functions or manipulating mathematical terms, is getting less important. The importance of calculus, which is a typical attribute in common education, loses ground to the importance of interpreting and demonstrating. By including real-life-problems, through the aspect of modeling, those aspects can be enforced with the help of technology. So the role of modeling in education is enforced and students can get aware of the enormous prominence of modeling in Mathematics education.

References

- Blum, W. (2005). "Modellieren im Unterricht mit der "Tanken"-Aufgabe". In: Mathematik lehren, H. 128, p. 18-21, Erhard Friedrich Verlag, Seelze.
- Fuchs, K.J.; Blum W. (2008). "Selbständiges Lernen im Mathematikunterricht mit, beziehungsreichen' Aufgaben". In: Thonhauser, J. (Hrsg.) Aufgaben als Katalysatoren von Lernprozessen (p. 135–148). Waxmann Verlag, Münster.
- BMUKK Lehrplan Mathematik (2004). AHS-Lehrplan Mathematik, BMUKK, Wien (http://www.bmukk.gv.at).
- Maaß, K. (2005a). "Modellieren im Mathematikunterricht der S I". In: JMD 26 (2), p. 114–142, Teubner Verlag, Wiesbaden.
- Maaß, K. (2005b). "Stau – eine Aufgabe für alle Jahrgänge!" In: PM Heft 47 (3), p. 8–13, Aulis Verlag Deubner, Köln.
- Meissner, A. (1996): "Gedanken zum Erwerb der grundlegenden Begriffe der Analysis unter Verwendung von Computern und grafikfähiger TR". MU, Heft 6, S. 42–59.
- Möhringer, J. (2006). Bildungstheoretische und entwicklungsadäquate Grundlagen als Kriterien für die Gestaltung von Mathematikunterricht am Gymnasium. Dissertation, LMU München.
- Schneider, E. (1999): "Analysisausbildung mit dem Computer". In: mathematica didactica, Heft 22, Bd. 1, S. 50–75.
- Siller, H.-St.; Maaß, J. (2008). "Fußball EM mit Sportwetten". In: Brinkmann, A.; Oldenburg, R.: ISTRON – Anwendung und Modellbildung im Mathematikunterricht, diVerlag franzbecker, Hildesheim (to appear in print).
- Deuber, R. (2005). "Lebenszyklus einer Weissblechdose – Vom Feinblech zur versandfertigen Dose". In: http://www.swisseduc.ch/chemie/wbd/docs/modul2.pdf, Baden.
- Vollrath, H.J. (2001): Grundlagen des Mathematikunterrichts in der Sekundarstufe, Spektrum Akademischer Verlag, Heidelberg, Berlin.

COGNICIÓN, PROTOFÍSICA Y LENGUAJE

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Resumen

Conformamos en nuestra mente combos de improntas, paquetes de experiencia que esperamos ver aparecer de ese modo y en ciertas secuencias recurrentes. Disponemos de "recetas" -en las que anidan esos *estigmas de mundo* encarnados- para hacer predecible el acaecer natural, pero usamos otras para representarnos las intenciones de los seres vivos y actuar en consecuencia. Los caprichos pertenecen al universo psicológico. Nos sorprende que la realidad no se ajuste al "programa". Por ello nos cuesta dar crédito a situaciones en las que vemos desagregados a aquellos combos o alteradas sus secuencias.

En este artículo describiremos algunos de esos condensados de experiencia. Mostraremos que ellos trascienden el ámbito del pensamiento donde estructuran nuestras ideas acerca del mundo y emergen en el lenguaje. Con metáforas como vectores se movilizan y aportan de ese modo soporte cognitivo para abarcar facetas más abstractas de nuestra realidad, desde la de nuestras volubles emociones hasta el esquivo concepto del tiempo.

Hay una física vitalmente eficaz pero precaria a la hora de abandonar la estrechez de nuestra experiencia directa. Aportaremos evidencia acerca de que la tensión entre estas nociones espontáneas y las que pone sobre la palestra la Física constituye un obstáculo para el aprendizaje de los conceptos científicos. El lenguaje nos dará una clave para entrar en el reino de la protofísica y operar sobre él.

Palabras clave

Enseñanza de las ciencias - Semántica cognitiva - Lingüística cognitiva - Metáfora - Física ingenua - Esquemas de imágenes.

Summary

Our minds are made up on sets of experience woven in a cognitive fabric. We expect those marks to appear secuenced that way again and again.

We have got models for reality to become predictible. Those models are built up from our bodies' activities in the world. Another set of action-formula is used to interact with our social enviroment where programs are more elusive than in the realm of objects and forecasts are tied to reading our mates gestures.

In this article we describe those packs which gather our ancestral behavior written in our flesh. We show that they explode from the kingdom of thought and emerge in language